On the Performance of TCM with Channel State Information in Frequency Flat Rayleigh Mobile Channels

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SUMMARY In this paper we analyze the performance of Trellis Coded Modulation (TCM) schemes with coherent detection operating in a frequency flat, mobile Rayleigh fading environment, and with different knowledge levels on both the amplitude and phase fading processes (the latter is not assumed as usual to be ideally tracked), or Channel State Information (CSI). For example, whereas ideal CSI means that both the amplitude and phase fading characteristics are perfectly known by the receiver, other situations that are treated consider perfect knowledge of the amplitude (or phase) with complete disregard of the phase (or amplitude), as well as non concerns on any of them. Since these are extreme cases, intermediate situations can be also defined to get extended bounds based on Chernoff which allow the phase errors, in either form of constant phase shifts or randomly distributed phase jitter, to be included in the upper bounds attainable by transfer function methods, and are applicable to multilevel signaling schemes. We found that when both fading characteristics are considered, the availability of CSI enhances significantly the performance. Furthermore, for non constant envelope schemes with non ideal CSI and for constant envelope schemes with phase errors, an asymmetry property of the pairwise error probability is identified. Theoretical and simulation results are shown in support of the analysis.

KEY WORDS: trellis coded modulation, channel state information, Rayleigh channel, Chernoff bound, phase ambiguities, phase jitter

1. Introduction

In recent years, the application of efficient coded modulation schemes [1] to channels other than the purely Gaussian have been subject of extensive research. Among these, the frequency flat (or non selective) mobile Rayleigh fading channel have deserved much attention because of its power and bandwidth practical restrictions [2]-[11].

On the other hand, in order to assist to the Viterbi decoding of the received signals, it is usual to extract information (Channel State Information (CSI)) about the amplitude and phase fading processes. This is an operation closely connected to that of carrier recovery. In the literature, it is sometimes assumed that the uniformly distributed phase fading is ideally tracked by a suitable circuit, and that only the amplitude in the form of a Rayleigh distributed random process remains affecting the received signal. Consequently, availability or unavailability of CSI is referred only to the amplitude fading. Under this criterion, it has been stated that few is the gain to be paid by ideal CSI [5]. Thus, all the bounds derived in Refs. [2]-[4] are based on this model, whereas in Ref. [10], both fading characteristics are considered through leading to so called truncated bounds, wherein only a subset of dominant terms are considered. Also in Ref. [11], the two characteristics are included in the analysis but the same is restricted to the case of no amplitude CSI, while a quasi analytical method is employed to obtain upper bounds.

The main purpose of this paper is to analyze the performance of TCM schemes in a system model where both the amplitude and the phase fading processes are considered. We analyze the following cases: perfect knowledge of the amplitude and phase fading, perfect knowledge only about the amplitude (or the phase) while the phase (or amplitude) is fully ignored, and also complete disregard of both. For the purpose of predicting the behavior of such schemes, we apply the Chernoff bound to the pairwise error probability. While the attainable bounds may in some circumstances be considered as weak, they are still the fast and simplest to calculate, being able to provide upper bounds by transfer function techniques given that an ideal interleaver/deinterleaver is used [2]-[4],[9]. We show then that the four above situations can be reduced to two in generalized expressions, which allow the attainment of extended bounds wherein phase errors in the form of constant phase shifts or randomly distributed phase jitter can be directly specified. The paper is organized as follows. Section 2 introduces the system model. In Sect. 3, following a brief review of the transfer function bounds, we discuss the system's performance based on the availability of CSI. The phase shifts and phase jitter phenomena are treated in Sect. 4. To illustrate the validity of the analysis, theoretical and simulation results are presented in Sect. 5, and finally our conclusions are resumed in Sect. 6.

2. System Model

As shown in Fig. 1, the system model consists, at the
transmitter side, of a convolutional encoder and a theoretically perfect block interleaver followed by a mapper which outputs a symbol on the basis of set partitioning rules. The block interleaver is used to counteract the correlation on successively faded symbols, yielding by the way a mathematically tractable model. The transmission media includes multiplicative fading as well as additive white Gaussian noise. Due to the displacement of the vehicle, the correlation evidences itself by shaping the power spectrum associated to the fading, resembling a filter with cutoff given by the maximum Doppler frequency $f_D$ [13].

Upon arrival, the signal is input to a demodulator, and also to a circuit for carrier recovery (for accomplishing coherent detection) and for estimating the channel (to provide CSI). The outputs denoted $z_n$ and $\xi_n$ are deinterleaved to restore the original sequence order, prior to their processing by the Viterbi decoder (assumed to include a quantizer for the aim of soft-decoding). These deinterleaved counterparts are represented by $z_n$ and $\xi_n$, respectively, and $z_n$ is given as

$$z_n = c_n s_n + \eta_n$$

(1)

where $c_n$ is a complex valued zero-mean stationary Gaussian random process, which characterizes the multipath fading channel. Accordingly, its Rayleigh distributed amplitude and uniformly distributed phase affect the transmitted complex valued symbol $s_n$, and the same is further disturbed by the zero-mean, also stationary Gaussian process $\eta_n = \eta_n + j\eta_n$, independent of $c_n(\eta_n$ and $\eta_n$ are independent identically distributed (i.i.d.) Gaussian processes, each with variance $\sigma^2$).

The fading channel coefficient is defined as

$$c_n = \rho_n e^{j\theta_n} = c_n + j\eta_n$$

(2)

and it follows that the non-linear transformations $c_n = \rho_n \cos \theta_n$ and $c_n = \rho_n \sin \theta_n$ yield i.i.d. Gaussian distributed processes.

Based on the availability of CSI, or on the knowledge of $\rho_n$ and $\theta_n$ by the receiver, we will treat in detail four (actually extreme) cases in the forthcoming section. Consideration of each extreme case has practical importance because they might occur if the circuits used to compensate $\rho_n$ and/or $\theta_n$ are broken down. Furthermore, they are theoretically important because the same can be used to verify the validity of the results for imperfect CSI treated in Sect. 4.

3. Error Probability Analysis

3.1 Review of Transfer Function Bounds

Let's recall the bit error probability upper bound given by

$$P_b \leq \frac{1}{mN} \left| \frac{\partial T(D, I) \cdot 1^T}{\partial I} \right|_{I=1}$$

(3)

where $m$ is the number of input bits, and $N$ is the number of convolutional encoder states [12]. Also, $T(D, I)$ is a $N \times N$ transfer function matrix given in terms of the parameter $D$ (to be defined later in connection with the Chernoff bound) and of the indeterminate $I$ (introduced to extend the capability of the transfer function in attaining bit error probability upper bounds). $1$ is an $N$-dimensional row vector with entries all equal to 1, and $1^T$ is its transpose.

From Ref. [12], we can write

$$T(D, I) = \sum_{\xi \in \mathcal{S}_I} \left( \sum_{\xi' \in \mathcal{S}_I} G(\xi, \xi') \right)$$

$$G(s_L, s_L') = \prod_{n=1}^{L} \sum_{n} \phi^T \left( e_n \right)$$

(4)

(5)

where $s_L$ and $s_{L'}$ denote the transmitted and decoded sequences, respectively, defining an error event of length $L$. In turn, the $N \times N$ matrix $G(s_L, s_{L'})$ contains as elements, error event probability upper bounds for a given pair of sequences $(s_L, s_{L'})$. $G(e_n)$ is referred to as the error weight matrix, associated to the error codeword $e_n$, assigned as a branch label to a particular transition in the $N$ state transition diagram of the underlying convolutional encoder [12]. Finally, the exponent of $I$, $e_n$ indicates the number of erroneous information bits in each transition. Computation of the upper bound in Eq. (3) by means of the matrix transfer function in Eq. (4) is particularly suited for non-uniform or error probability sequence dependent schemes, though it may be simplified to a scalar calculation when certain uniformity conditions are satisfied [12]. In any case, the $p$-th row, $q$-th column entry of the error weight matrix is obtained by

$$[G(e_n)]_{p,q} = \frac{1}{2^N} D = \frac{1}{2^N} E[\exp \lambda \delta_n]$$

(6)

where the operator $E[\cdot]$ denotes statistical expectation. $E[\exp \lambda \delta_n]$ is referred as Chernoff factor hereafter [4], and originates from the Chernoff bound (with $\lambda$ as a parameter to be optimized) on the pairwise error probability $P[S_L \rightarrow S_L']$, as follows
\[ P[S_i \rightarrow S_j] = \Pr \left[ \sum_{n=1}^{L} \delta_n > 0 \right] \leq \frac{1}{2} \prod_{n=1}^{N} E[\exp \lambda \delta_n] \]  

where \( |\beta_n|^2 \) and \( |\beta'_n|^2 \) are the squared distance branch metrics of \( S_i \) and \( S_j \), respectively, under the assumption that \( S_i \) is a correct path. The one half factor in Eq. (7) tightens the Chernoff bound according to Ref. [16], provided that the underlying scheme is uniform. Furthermore, \( \beta_n \) and \( \beta'_n \) can be represented as

\[ \begin{align*}
\beta_n &= c_n \delta_n + \frac{\eta_n - \xi_n \delta_n}{2} \\
\beta'_n &= c_n \delta_n + \frac{\eta_n - \xi_n \delta_n}{2}
\end{align*} \]  

where depending on the CSI, \( \xi_n \) may assume four limiting situations which we analyze next.

### 3.2 Perfect Knowledge of Both \( \rho_n \) and \( \theta_n \) (\( \xi_n = \rho_n e^{j\theta_n} \))

In this case, both the amplitude \( \rho_n \) and the phase fading \( \theta_n \) processes are perfectly estimated, deserving the denomination of ideal CSI. Thus, the branch metrics \( \beta_n \) and \( \beta'_n \) become

\[ \begin{align*}
\beta_n &= c_n \delta_n + \eta_n - c_n \delta_n \eta_n \\
\beta'_n &= c_n \delta_n + \eta_n - c_n \delta_n \eta_n
\end{align*} \]  

where \( \eta_n = s_n - s'_n = e_n e_n^* + j e_n e_n \) is the complex valued branch symbol error. With the previous definitions, \( \delta_n \) is obtained as

\[ \begin{align*}
\delta_n &= -\rho_n d_n^* - 2 \rho_n (\cos \theta_n e_n - \sin \theta_n e_n) \eta_n \\
&- 2 \rho_n (\cos \theta_n e_n^* + \sin \theta_n e_n) \eta_n
\end{align*} \]  

where \( d_n^2 = e_n^* + e_n^* \) is the branch symbol squared Euclidean distance. A usual way of obtaining the Chernoff factor of Eq. (7) is to average \( \delta_n \) in Eq. (13) over \( \eta_n \) conditioned on both \( \rho_n \) and \( \theta_n \), and then to average successively over the two laters to remove the conditions. (In the case of ideal phase tracking, e.g. [2], [3], the only condition is on \( \rho_n \).

Here, we show an alternative way of looking at \( \delta_n \) as a single random process by investigating its probability density function. The convenience of this method is that the Chernoff factor can be obtained directly in one step, thus avoiding the need of successively averaging over \( \eta_n \), \( \rho_n \) and \( \theta_n \). We define \( X_n = c_n e_n + j X_n^e \) (in Appendix A, it is shown that \( X_n \), and \( X_n^e \) are mutually independent processes with variances \( \sigma^2 = \sigma^2 = \sigma^2 \)), and replace Eqs. (11), (12) in Eq. (8) to get

\[ \begin{align*}
\delta_n &= |X_n|^2 - 2 \Re \{X_n^e \eta_n\} \\
&= -X_n (X_n + 2 \eta_n) - X_n^e (X_n^e + 2 \eta_n)
\end{align*} \]  

where \( X_n^* \) stands for the complex conjugate of \( X_n \). We can easily show that \( \delta_n \) and \( \delta_n^* \) are statistically independent.

Let now

\[ \delta_n \sim N \left(X_n^e (2 \eta_n) + X_n, \sigma^2 \right) \]  

where \( \sigma^2 \) represents either \( \sigma^2 \) or \( \sigma^2 \). By making \( \phi_n = X_n + 2 \eta_n \), we get

\[ f(\phi_n) = \frac{1}{\sqrt{2 \pi} \sigma_n} \exp \left( -\frac{\phi_n^2}{2 \sigma_n^2} \right) \]  

which is a zero-mean Gaussian random process with variance \( \sigma_n^2 = \sigma_k^2 + 4 \sigma^2 \). Then, from Eq. (15), \( \delta_n \) is a product of two statistically dependent Gaussian distributed random processes \( X_n^e \) and \( \eta_n \).

We deal now with the derivation of the probability density function (pdf) of \( \phi_n > \phi_n \). In principle, we use the result

\[ f(\phi_n) = \frac{\eta_n}{\phi_n} \exp \left[ -\frac{\phi_n}{\phi_n^2} \right] \]  

where \( \eta_n = \phi_n / \phi_n \) is the joint pdf of \( X_n^e \) and \( \phi_n \). By joint normality we obtain

\[ g(\phi_n, \psi_n) = \frac{1}{2 \pi \sigma_n \sigma_\phi} \exp \left[ -\frac{1}{2 (1 - r_{xy})} \left( \frac{X_n^e - 2 \phi_n}{\sigma_n^2} - \phi_n / \sigma_\phi \right) + \frac{\phi_n^2}{2 \sigma_\phi^2} \right] \]  

where \( r_{xy} \) is the correlation coefficient defined as

\[ r_{xy} = \frac{\sigma_n}{\sigma_n \sigma_\phi} \]  

where the last equality holds since both \( X_n^e \) and \( \phi_n \) are zero-mean (\( m_n = 0 \) and \( m_n^e = 0 \)) and \( \sigma_n^2 = E[X_n^e] \).

Upon simplification Eq. (17) results

\[ f(\phi_n) = \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2 (1 - r_{xy})} \left( \phi_n - 2 \phi_n \frac{\sigma_n^2}{\sigma_\phi} + \phi_n^2 \frac{\sigma_n^2}{\sigma_\phi^2} \right) \right] \]  

\[ d\phi_n \]  

\[ = \frac{\exp \left[ -\frac{r_{xy} \sigma_n^2}{\sigma_\phi} + \frac{\sigma_n^2}{\sigma_\phi^2} \right]}{\pi \sigma_n \sigma_\phi \sqrt{1 - r_{xy}^2}} \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2 (1 - r_{xy})} \left( \phi_n^2 \frac{\sigma_n^2}{\sigma_\phi^2} + \phi_n^2 \frac{\sigma_n^2}{\sigma_\phi^2} \right) \right] d\phi_n \]  

where the property \( g(x, y) = g(-x, -y) \) is used. Let
\[ \tau_n = \psi_n \] such that \( d_{\tau_n} = 2 \phi \psi_n d\psi \). Thus
\[
(a) = \int_0^\infty \frac{1}{\psi_n} \exp \left[ -\frac{1}{2} \frac{\tau_n}{1-r_{\lambda}^2} \left( \frac{\tau_n}{1-r_{\lambda}^2} \right) \right] \\
\times \frac{1}{2\psi_n} d\tau_n \\
= \frac{1}{2} \int_0^\infty e^{-\rho \tau_n e^{-\lambda t}} d\tau_n
\]
where \( \rho = 1 \). (21)

We note that
\[
\int_0^\infty \frac{1}{\tau_n} e^{-\rho \tau_n e^{-\lambda t}} d\tau_n = \mathcal{L}[\tau_n e^{-\lambda t}] \\
\]
where \( \mathcal{L} \) denotes Laplace transform. Now in Ref. [15] it can be found
\[
\mathcal{L}[\tau_n e^{-\lambda t}] = \frac{1}{\lambda} \frac{\tau_n}{1-r_{\lambda}^2} \\
\]
which yields the bound as a function of a single parameter, the correlation coefficient \( r_{\lambda} \).

By replacing Eq. (24) in Eq. (20) we obtain
\[
f (\delta_{\theta_n}) = \frac{\exp \left[ \frac{r_{\theta \phi} \delta_{\theta_n}}{(1-r_{\lambda}^2) \sigma_x \sigma_y} \right]}{(1-r_{\lambda}^2) \sigma_x \sigma_y} \left( \frac{\delta_{\theta_n}}{1-r_{\lambda}^2} \right)
\]
where \( -\infty < \delta_{\theta_n} < \infty \). In Appendix B it is demonstrated that \( f (\delta_{\theta_n}) \) of Eq. (25) satisfies the probability density function definition (i.e., that \( \int f (\delta_{\theta_n}) d\delta_{\theta_n} = 1 \)).

3.3 Perfect Knowledge of \( \rho_n \). Phase \( \theta_n \) Ignored (\( \xi_n = \rho_n \))

Here, while the amplitude \( \rho_n \) is exactly estimated, no information on the phase fading process \( \theta_n \) is available, and consequently no compensation of the phase fading effect on the received signal is possible. In this case, we have
\[
\beta_n = \gamma_n + \eta_n - \rho_n \theta_n \\
\beta_n^* = \gamma_n + \eta_n - \rho_n \theta_n
\]
which leads to
\[
\delta_n = \left( |s_n|^2 - |\alpha_n|^2 \right) \rho_n^2 - 2 \Re \{ (c_n \gamma_n + \eta_n \theta_n) \} \\
= k_n \rho_n^2 + (a_n \cos \theta_n + b_n \sin \theta_n) \rho_n + \rho_n \eta_n \\
k_n = |s_n|^2 - |\alpha_n|^2 \\
a_n = -2 (s_n \eta_n \theta_n + s_n \theta_n) \\
b_n = -2 (s_n \eta_n \theta_n - s_n \theta_n) \\
c_n = -2 \eta_n \theta_n - 2 \eta_n \theta_n
\]
Note that $k_n$ represents the energy difference between the correct and incorrect signals ($k_n=0$ for constant envelope signaling), and can be shown to satisfy the following relation

$$k_n = -d_n^2 + 2 \Re \{ s_n^* e_n \} = -d_n^2 - a_n$$ (39)

In the near past, employing non constant envelope schemes (e.g. QAM) in fading channels was judged as prohibitive, because of their requirements of very precise fading compensation techniques [5]. Recently however, several developments have led to a change of this mentality, and in particular QAM schemes are seen as very promising due their high spectral efficiency [17], [18].

In the current case, it is not possible to obtain an explicit expression on the probability density function of $\theta_n$. However, it should be noted that to simplify the subsequent expectation, we have defined the Gaussian process $\alpha_n$ consisting of a linear transformation of the mutually independent and Gaussian distributed processes $\eta_{kn}, \eta_{kn}$. The variance of $\alpha_n$ is obtained as

$$\sigma_n^2 = 4d_n^2\sigma^2$$ (40)

Then, by successively averaging over $\alpha_n$ and $r_n$, we get

$$E[\exp \lambda \delta_n | \theta, a_n] = \frac{1}{1 - \frac{\lambda}{2\sigma^2} [k_n + a_n \cos \theta_n + b_n \sin \theta_n + \lambda d_n^2]}$$ (41)

where optimization of $\lambda$ does not lead to a time independent quantity. Thus, it must be numerically optimized (to minimize the upper bound of Eq. (3) ). Finally, Eq. (41) must be averaged over the random phase $\theta_n$, which is uniformly distributed between $-\pi$ and $\pi$. Thus, we write

$$E[\exp \lambda \delta_n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} E \{ \exp \lambda \delta_n \} d\theta_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{1 - \frac{\lambda}{2\sigma^2} \cos \theta_n + \lambda d_n^2} d\theta_n$$ (42)

where $\nu_e = -\lambda d_n^2/2\sigma^2$, $\nu_n = -\lambda d_n^2/2\sigma^2$ and $x_n = 1 - (\lambda/2\sigma^2)(k_n + \lambda d_n^2)$. The above integral is available from tables for the undefined limits case. For our case, the following solution is gotten

$$E[\exp \lambda \delta_n] = \frac{1}{\sqrt{x_n^2 - \nu_n^2 - w_n^2}} \left( \frac{1}{\sqrt{1 - \frac{x_n^2}{2\sigma^2}(k_n + \lambda d_n^2)^2}} - \frac{\nu_n^2 x_n^2 d_n^2}{\sigma^2} \right)$$ (43)

restricted to the condition $x_n^2 > \nu_n^2 + w_n^2$.

3.4 Perfect Knowledge of $\theta_n$, Unknown Amplitude $r_n$ ($\xi = e^{j\theta_n}$)

We consider now the case where the phase fading process $\theta_n$ is ideally tracked, whereas the amplitude $r_n$ is by no means compensated.

The branch metrics under the present condition become

$$\beta_n = c_n s_n + \eta_n - e^{j\alpha_n} s_n$$ (44)

$$\beta_n' = c_n s_n + \eta_n - e^{j\alpha_n} s_n'$$ (45)

and after some simplification

$$\beta_n = |s_n|^2 - |s_n'|^2 - 2 \Re \{ (c_n s_n + \eta_n) e^{j\alpha_n} e_n \}$$

$$= k_n + a_n r_n + g(\theta_n) \eta_n + h(\theta_n) \eta_n$$ (46)

where $k_n$ and $a_n$ are the same as previously defined, and

$$g(\theta_n) = -2(\cos \theta_n e_n - \sin \theta_n e_n)$$ (47)

$$h(\theta_n) = -2(\cos \theta_n e_n + \sin \theta_n e_n)$$ (48)

Chernoff bounding the pairwise error probability produces the result

$$E[\exp \lambda \delta_n] = e^{\frac{\lambda k_n}{2\sigma^2}} e^{\frac{\lambda^2 d_n^2}{2\sigma^2}} \left[ 1 + \frac{\lambda a_n}{2\sigma^2} \right] Q(-\frac{\lambda}{2\sigma^2} a_n)$$ (49)

where $Q(\cdot)$ denotes the tail integral of the Gaussian density, also called $Q$ function, defined by $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-t^2/2} dt$. Once more $\lambda$ demands a numerical optimization. The result of Eq. (49) also coincides with that of Ref. [3] (Eq. (19)), which treats only the particular case $k_n=0$ under the Rayleigh rather than the Gaussian fading model. Thus, the assumption of perfect phase tracking plus Rayleigh amplitude fading unknown to the receiver is tantamount to assuming that the fading is Gaussian modeled from where only the phase is perfectly known.

3.5 Unknown Both Amplitude $r_n$ and Phase $\theta_n$ ($\xi = e^{j\theta_n}$)

This is the situation that arises when no information on the fading channel characteristics is used in assisting the Viterbi decoding. For the present condition

$$\beta_n = c_n s_n + \eta_n - s_n$$ (50)

$$\beta_n' = c_n s_n + \eta_n - s_n'$$ (51)

and also

$$\delta_n = (|s_n|^2 - |s_n'|^2) - 2 \Re \{ (c_n s_n + \eta_n) e_n \}$$

$$= k_n + a_n$$ (52)
\[ a_n = a_{\alpha} c_{\alpha} + b_{\alpha} c_{\alpha} + 2 \epsilon_{\alpha} \eta_{\alpha} - 2 \epsilon_{\alpha} \eta_{\alpha} \]  

(53)

where \( a_{\alpha} \) is now given by the linear transformation of the mutually independent and Gaussian distributed processes \( c_{\alpha}, c_{\alpha}, \eta_{\alpha}, \eta_{\alpha}, \) and \( a_{\alpha}, b_{\alpha} \) are the same defined before. Then, \( \delta_{\alpha} \) is a \( k_{\alpha} \)-mean normal process such that for constant envelope schemes, i.e., \( k_{\alpha} = 0 \), it results zero-mean. In this case, \( \sum_{n=1}^{\infty} \delta_{\alpha} \) becomes as well zero mean \( \left( \sum_{n=1}^{\infty} k_{\alpha} = 0 \right) \) Gaussian distributed and consequently

\[ \Pr \left[ \sum_{n=1}^{\infty} \delta_{\alpha} > 0 \right] = \frac{1}{2} \]  

(54)

which renders a useless system's performance irrespective of the complexity of the underlying code. Thus, any pretension of recovering good quality signals disestimating the use of CSI becomes nonsense. Or alternatively stated, much is the noise immunity that can be gained if CSI is employed.

In the non constant envelope case like QAM, the pairwise error probability will depend on the sign of \( \sum_{n=1}^{\infty} k_{\alpha} \), resulting larger or less than 0.5 if the same is positive or negative, respectively. Since \( \sum_{n=1}^{\infty} \delta_{\alpha} \) is Gaussian distributed, the exact probability value can be obtained by means of the Q function.

The Chernoff factor for \( k_{\alpha} = 0 \) is obtained as follows

\[ E[\exp \lambda \delta_{\alpha}] = \exp \left[ \frac{k_{\alpha} \lambda^2}{2 \sigma^2} \right] \exp \left[ \frac{\lambda^2 (\sigma^2 + 1)}{2 \sigma^2} \right] \]  

(55)

and once more numerical optimization of \( \lambda \) is required (for \( k_{\alpha} = 0 \) it can be shown that \( \lambda_{opt} = 0 \)).

At this point, it is pertinent to point out that for the ideal CSI case of Sect. 3.1, the Chernoff factor and accordingly the pairwise error probability upper bound for any signaling scheme depends only on the squared branch distance \( d_2^2 \) (see Eq. (31)). On the other hand, the non-ideal CSI Chernoff factors given by Eqs. (43), (49) and (55) show not only dependency on \( d_2^2 \) but also on \( k_{\alpha} \) (note that when \( k_{\alpha} = 0, |s_\alpha|^2 \) in Eqs. (43) and (31) is not a constant). This fact reveals a sort of asymmetry of the pairwise error probability for non constant envelope schemes with non ideal CSI since, by exchanging the correct symbol \( s_{\alpha} \) with the incorrect \( \tilde{s}_{\alpha} \) \( k_{\alpha} \) becomes multiplied by \(-1\) and this leads to a different value of the Chernoff factor. In other words, for a given pair of sequences \((S_\alpha, \tilde{S}_\alpha)\), the probability that \( S_\alpha \) is decoded provided that \( S_\alpha \) was transmitted is not equal to the probability that \( S_\alpha \) is erroneously favored against a hypothetically correct \( S_\alpha \). Thus, in the non ideal CSI context the system results non uniform or sequence dependent [12]. As a consequence, the condition required for the one half factor in Eq. (7) to hold is no longer satisfied, and one must drop it when computing the corresponding bounds.

In the next section the four cases analyzed above are reduced into two, and extended Chernoff bounds are derived by means of which other sources of performance degradation such as phase shifts or phase jitter can be taken into account.

4. Phase Shift and Phase Jitter

In this section, we consider that the phase fading is imperfectly known (rather than perfectly known or totally unknown) to the receiver, while the amplitude fading may be exactly estimated or on the contrary fully ignored. This permits the inclusion of additional sources of performance degradation in the theoretical analysis, explicitly, of constant phase shifts and phase jitter, which occur in practice attempting against coherent detection. This leads us to the two cases which we treat next.

4.1 Perfect Knowledge of \( \rho_{\alpha} \), Imperfect Phase Knowledge \((\tilde{\xi}_{\alpha} = \rho_{\alpha} e^{i\xi_{\alpha}})\)

The branch metrics become

\[ \beta_{\alpha} = c_{\alpha} s_{\alpha} + \eta_{\alpha} - \rho_{\alpha} e^{i\xi_{\alpha}} s_{\alpha} \]  

(56)

\[ \beta_{\alpha} = c_{\alpha} s_{\alpha} + \eta_{\alpha} - \rho_{\alpha} e^{i\xi_{\alpha}} s_{\alpha} \]  

(57)

and it follows that

\[ \delta_{\alpha} = (|s_{\alpha}|^2 - |s_{\alpha}|^2) \rho_{\alpha} - 2 \rho_{\alpha} R\{ (s_{\alpha} \eta_{\alpha} + \rho_{\alpha} e^{i\xi_{\alpha}} s_{\alpha}) \} \]  

(58)

where \( k_{\alpha} \) is the same as defined before, and

\[ e_{\alpha}(\tilde{\theta}_{\alpha}, \tilde{\theta}_{\alpha}) = a_{\alpha} \cos \tilde{\theta}_{\alpha} + b_{\alpha} \sin \tilde{\theta}_{\alpha} \]  

(59)

\[ a_{\alpha} = -2(s_{\alpha} e_{\alpha} + s_{\alpha} e_{\alpha}) \]  

(60)

\[ b_{\alpha} = -2(s_{\alpha} e_{\alpha} - s_{\alpha} e_{\alpha}) \]  

(61)

\[ \theta_{\alpha} = \theta_{\alpha} - \tilde{\theta}_{\alpha} \]  

(62)

\[ a(\tilde{\theta}_{\alpha}) = g(\tilde{\theta}_{\alpha}) \eta_{\alpha} + h(\tilde{\theta}_{\alpha}) \eta_{\alpha} \]  

(63)

\[ g(\tilde{\theta}_{\alpha}) = -2(\cos \tilde{\theta}_{\alpha} \eta_{\alpha} - \sin \tilde{\theta}_{\alpha} \eta_{\alpha}) \]  

(64)

\[ h(\tilde{\theta}_{\alpha}) = -2(\cos \tilde{\theta}_{\alpha} \eta_{\alpha} + \sin \tilde{\theta}_{\alpha} \eta_{\alpha}) \]  

(65)

We can further rewrite \( e_{\alpha}(\tilde{\theta}_{\alpha}, \tilde{\theta}_{\alpha}) \) by noting that

\[ s_{\alpha}^2 s_{\alpha} - s_{\alpha}^2 = (s_{\alpha} e_{\alpha} + s_{\alpha} e_{\alpha}) + j(s_{\alpha} e_{\alpha} - s_{\alpha} e_{\alpha}) \]  

(66)

from where \( a_{\alpha} \) and \( b_{\alpha} \) can be identified as

\[ a_{\alpha} = -2 \Re \{s_{\alpha} - s_{\alpha}\} \]  

(67)

\[ b_{\alpha} = -2 \Im \{s_{\alpha} - s_{\alpha}\} \]  

(68)

which obviously means that \( a_{\alpha} \) and \( b_{\alpha} \) are in quadrature phase. Defining an angle, say \( \phi_{\alpha} \), satisfying \( \phi_{\alpha} \)
\[ a_n = \sqrt{a_n^2 + b_n^2} \cos \phi_n \]
\[ b_n = \sqrt{a_n^2 + b_n^2} \sin \phi_n \]

and replace Eqs. (69) and (70) in Eq. (59) to obtain

\[ e_n(\theta_{dn}) = \sqrt{a_n^2 + b_n^2} \cos (\theta_{dn} - \phi_n) = 2|s_n^x - s_n^y| \cos (\theta_{dn} - \phi_n) \]
\[ = 2|s_n^x| d_n \cos (\theta_{dn} - \phi_n - \phi_n) \]

where Eq. (71) follows from Eqs. (67) and (68), and Eq. (72) comes from noting that \[ |s_n^x| s_n^y = |s_n^x||e_n| = |s_n| d_n. \] Also, we have changed \( e_n(\theta_{dn}) \) by \( e_n(\theta_{cn}) \) since, as can be seen from Eq. (59), the former is already a function of the phase difference \( \theta_{dn} \). This phase difference may be a constant quantity (a phase shift in practice), or also a random variable (a phase jitter or noisy phase reference). We will consider this shortly.

The Chernoff factor for the present case (conditioned on \( \theta_{dn} \)) can be shown to be

\[ E[\exp \lambda d_n|\theta_{dn}] = \frac{1}{1 - \frac{\lambda}{2\sigma^2} \left[ k_n + 2|s_n|d_n \cos (\theta_{dn} - \phi_n) + \lambda d_n^2 \right]} \]

which is directly applicable for the case of constant phase shift, by replacing the given phase shift value in place of \( \theta_{dn} \).

On the other hand, when the analysis considers degradation by a noisy phase reference, the above equation must be averaged over a randomly distributed variable \( \theta_{dn} \). As in Refs. [19], [20], we assume that \( \theta_{dn} \) is zero-mean, Gaussian distributed, and with variance \( \sigma_{\theta_{dn}} \) such that

\[ p(\theta_{dn}) = \frac{1}{\sqrt{2\pi \sigma_{\theta_{dn}}}} \exp \left[ -\frac{\theta_{dn}^2}{2\sigma_{\theta_{dn}}^2} \right]. \]

For the particular case where \( \theta_{dn} = 0 \), \( \theta_{cn} = \theta_{cn} \), it can be seen from Eq. (59) that \( e_n(\theta_{dn}) \) reduces to \( a_n \). By replacing Eq. (39) in Eq. (73) and averaging over \( \theta_{dn} \), (whose probability density function is now a unit area impulse defined by \( p(\theta_{dn}) = \delta(\theta_{dn}) \)), we arrive (with \( \lambda_{avg} = 1/4\sigma^2 \)) to the same expression given by Eq. (31). Similarly, for \( \theta_{dn} = \theta_n \) or \( \theta_{dn} = 0 \), we arrive to the case described in Sect. 3.2. Of course, in any of these two limiting cases, it is preferable to use Eq. (31) or Eq. (43).

4.2 Complete Unknowledge of \( \rho_n \), Imperfect Phase Knowledge (\( \xi_n = e^{\phi_n} \))

Here we have

\[ \beta_n = c_n s_n + \eta_n - e^{\phi_n} a_n s_n \]
\[ \beta_n' = c_n s_n + \eta_n - e^{\phi_n} a_n' s_n' \]

and \( \delta_n \) is given by

\[ \delta_n = \left( |s_n|^2 - |s_n'|^2 \right) - 2 \Re \{ (c_n s_n + \eta_n) \ast e^{\phi \xi_n} e_n \}
\]

\[ = k_n + e_n(\theta_{dn}) \rho_n + a_n(\theta_{cn}) \]

with \( k_n \) and \( a_n(\theta_{cn}) \) as already defined. By successively averaging over \( a_n(\theta_{cn}) \) (a Gaussian process) and over \( \rho_n \), the Chernoff factor becomes

\[ E[\exp \lambda d_n|\theta_{dn}] = \exp \frac{\lambda d_n^2}{2\sigma^2} \cdot \frac{1 + \lambda d_n^2}{2\sigma^2} \cdot Q\left( \frac{\lambda d_n}{\sqrt{2}} \right) \]

where \( Q(\cdot) \) is the complementary error function.

5. Results

In this section we present some numerical results. In Fig. 2, the error event probability for one path of the 8

![Fig. 2 One path pairwise error probability (8 state 16 QAM).](image-url)
state 16 QAM Du-Vucetic code is given, in order to illustrate the asymmetry property of the error event probability for non constant envelope schemes, as analyzed in Sect. 3. This feedback code has been specially designed for the fading channel under the assumption of ideal CSI, and is specified by the octal coefficients $b_1=16$, $b_2=04$, $b_3=07$, $b_4=13$ [7]. The arbitrarily selected error event is defined by an assumed correct sequence $(s_{k_1}=1, s_{i_1}=1)$, $(s_{k_2}=1, s_{i_2}=1)$ and by an incorrect one given by $(s_{k'_3}=3, s_{i'_3}=3)$, $(s_{k''_3}=-3, s_{i''_3}=-3)$. The notation (+) and (−) stands for the original correct sequence and for its permutation, respectively. We must stress here that while for the ideal CSI case, or $ξ_α=ρ_αe^{jα_α}$, the optimum value of $λ = 1/4α^2$ is the same for one path and for the ensemble of paths which conform the union bound, this is not valid for the non ideal cases shown. That is, the optimum $λ$ obtained numerically to minimize the Chernoff bounded error event probability for the selected path is not the same than the one which would be obtained to minimize the union bound. This explains why it seems that non ideal CSI is doing better than perfect CSI. The asymmetry property for non ideal CSI is apparent, since permutation of the correct and incorrect sequences give different error event probability results. For $ξ_α=1$ and $ξ_α=ρ_α$, the loosest Chernoff bound for the (−) sequence is gotten (corresponding to $λ_{app}=0$). Since in the former case the exact pairwise probability can be computed also (as pointed out in Sect. 3.5) by means of the $Q$ function, we show those results with dotted lines.

The figures that follow next show both theoretical and simulation results, appearing in corresponding order. In Fig. 3, the 2/3 rate 8 state 8 PSK Ungerboeck code is evaluated for several values of phase shift, and for two conditions of CSI. As expected, the performance of the system worsens when the amplitude fading process is unknown to the receiver. However, since such process can only dilate or contract a signal vector without taking it out from its decision region, the associated degradation does not reach higher levels. Furthermore, the performance of the same above Du-Vucetic code under given phase shift values is shown in Fig. 4. In this case, the performance is negatively affected by a constant phase shift. The theoretical upper bounds for the condition $ξ_α=ρ_αe^{jα_α}$ are not shown since the degradation suffered in this case is too high for them to be considered useful. To corroborate this we show the result of the simulation when the phase error $θ_{th}$ is zero or equivalently $ξ_α=e^{jθ_{th}}$. It should be noted that a multilevel scheme as QAM is quite sensitive to the amplitude fading, since in this case the signal vector can easily reach a neighbor's decision region. Finally, assessment is made of the Ungerboeck code when failure in tracking the phase fading is noisy, and for two different CSI conditions. The performance of that scheme shown in Fig. 5, is seen degraded for the given phase standard deviation.

Although in all cases our theoretical results follow
the tendency of the simulations, there is a difference
between them which is typically observed when the
Chernoff bound is applied to fading channels. Rea-
sons behind are the non maximum likelihood metric
used for non ideal CSI [9] and also the parameter λ
which in such cases is optimized numerically for the
whole ensemble of error events. We have also seen that
the one half factor cannot always be used to tighten the
bounds. However, in combination with transfer func-
tion techniques the Chernoff bound is a fast and simple
way of providing error probability upper bounds.

6. Conclusions

We have analyzed the performance of TCM systems
in frequency flat mobile Rayleigh fading channels, by
considering the availability of Channel State Infor-
mation (CSI). First, we analyzed four cases which
can be seen as limiting situations. We have found that
CSI enhances significantly the system's performance
when both fading characteristics are considered. Also,
an asymmetry property of the pairwise error proba-
Bility has been identified for non constant envelope
schemes with non ideal CSI and for constant envelope
schemes with phase errors. Furthermore, extended
expressions of the Chernoff factors were obtained
where it is possible to directly specify phase shift values
or Gaussian distributed phase jitter variances, thus
yielding more useful Chernoff bound based upper
bounds on the bit error probability.

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Appendix A

From the definition of $X_n$, we have
\[
\bar{X}_n = e_{n} e_{n-1} - e_{n} e_{n} \tag{A.1}
\]
\[
X_n = e_{n} e_{n} + c_{n} e_{n} \tag{A.2}
\]
Then, the covariance is given as follows
\[
E[(X_n - \bar{X}_n)(X_n - \bar{X}_n)] = E[X_n X_n] \tag{A.3}
\]
\[
= e_{n} E[c_{n} e_{n}] - e_{n} e_{n} E[e_{n}] + e_{n} e_{n} E[e_{n}] \tag{A.4}
\]
\[ e_{l_{k}}^{'0} = e_{k_{s}}e_{i_{n}} \sigma_{i_{n}}^{2} + e_{k_{s}}e_{l_{k}} \sigma_{l_{k}}^{2} - e_{l_{k}}^{'0} = 0 \quad (A \cdot 5) \]

where Eq. (A-3) follows from considering that \( c_{k_{s}} \) and \( c_{l_{k}} \) are zero-mean. Equation (A-4) uses the definitions of \( X_{k_{s}} \) and \( X_{l_{k}} \), while Eq. (A-5) is obtained from the uncorrelatedness of the real and imaginary parts of \( c_{k_{s}} \), and from the fact that their respective variances are equal \( \sigma_{k_{s}}^{2} = \sigma_{l_{k}}^{2} = \sigma_{c}^{2}/2 \) [13]. Thus, \( X_{k_{s}} \) and \( X_{l_{k}} \) are not correlated. Now, we obtain the variances of \( X_{k_{s}} \) and \( X_{l_{k}} \) as

\[ \sigma_{k_{s}}^{2} = e_{k_{s}}^{2} \sigma_{c}^{2} + e_{i_{n}}^{2} \sigma_{s}^{2} \quad (A \cdot 6) \]
\[ \sigma_{l_{k}}^{2} = e_{l_{k}}^{2} \sigma_{c}^{2} + e_{l_{k}}^{2} \sigma_{s}^{2} \quad (A \cdot 7) \]
\[ \sigma_{k_{s}}^{2} = \sigma_{l_{k}}^{2} = \sigma_{s}^{2} = \frac{d^{2} \sigma_{c}^{2}}{2}. \quad (A \cdot 8) \]

### Appendix B

By splitting Eq. (25) in two terms and integrating over \( \delta w_{w} \),

\[ \int_{-\infty}^{\infty} f(\delta w_{w}) d\delta w_{w} = \frac{\sigma_{\delta_{\delta}}(1 - r_{\delta_{\delta}}^{2})}{\pi \sigma_{\delta_{\delta}} \sqrt{1 - r_{\delta_{\delta}}^{2}}} \]
\[ \cdot \left( \int_{0}^{\infty} e^{-wxw} K_{0}[w^{-}] dw^{-} + \int_{0}^{\infty} e^{wxw} K_{0}[w^{+}] dw^{+} \right) \quad (A \cdot 9) \]

where \( w^{-} = \delta w_{w}/(1 - r_{\delta_{\delta}}^{2}) \) and \( w^{+} = \delta w_{w}/(1 - r_{\delta_{\delta}}^{2}) \).

The integrals in Eq. (A-9) can be solved also by Laplace transformation [15] and by using the identity arcsec(x) + arcsec(-x) = \( \pi \)

\[ \int_{-\infty}^{\infty} f(\delta w_{w}) d\delta w_{w} = \frac{\sigma_{\delta_{\delta}}(1 - r_{\delta_{\delta}}^{2})}{\pi \sigma_{\delta_{\delta}} \sqrt{1 - r_{\delta_{\delta}}^{2}}} \cdot \frac{1}{\sqrt{1 - r_{\delta_{\delta}}^{2}}} \]
\[ \cdot (\text{arcsec}(r_{\delta_{\delta}}) + \text{arcsec}(-r_{\delta_{\delta}})) \]
\[ = 1. \quad (A \cdot 10) \]

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